

# Delta-N formalism for the evolution of the curvature perturbations in generalized multi-field inflation

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## Abstract

The  $\delta N$  formalism is considered to calculate the evolution of the curvature perturbation in generalized multi-field inflation models. The result is consistent with the usual calculation of the standard kinetic term. For the calculation of the generalized kinetic term, we improved the definition of the adiabatic field. Our calculation improves the usual calculation of  $\dot{\mathcal{R}}$  based on the field equations and the perturbations, giving a very simple and intuitive argument for the evolution equations in terms of the perturbations of the inflaton velocity. Significance of non-equilibrium corrections are also discussed, which is caused by the small-scale (decaying) inhomogeneities. This formalism based on the modulated inflation scenario (i.e., calculation based on the perturbations related to the inflaton velocity) provides a powerful tool for investigating the signature of moduli that may appear in string theory.

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# 1 Introduction

Inflation has become a major paradigm for explaining the very early Universe that is consistent with the observations, and current observations of the temperature anisotropy of the Cosmic Microwave Background (CMB) support the scale-invariant and Gaussian spectrum that is expected from the standard inflation scenario. However, there are some anomalies in the spectrum, such as a small departure from the exact scale-invariance or a certain non-Gaussian character[1], which are expected to reveal the dynamics of the fields that are responsible for the inflation. An obvious example is the observation of a small shift in the spectrum index  $n - 1 \neq 0$ , which suggests that there is a small departure from the scale-invariant evolution and which has been used to constrain the inflation potential [2]. More recently, it has been claimed that observations may support a significant non-Gaussian character in the spectrum [3]. For example, with regard to the generation of the anomaly that captures the inflation dynamics, there are models for inflation in which the spectrum is generated (1) during inflation [4, 5, 6, 7, 8], (2) at the end of inflation [9, 10, 11], (3) after inflation by preheating and reheating [12, 13, 14, 15, 16] or (4) by the curvatures [17, 18, 19, 20]. In addition to these scenarios, an inhomogeneous phase transition[21] may play crucial role in generating cosmological perturbations. In this paper, the first possibility is considered; the correction during inflation causes a significant anomaly in the spectrum. We have a special interest in the effects of massless excitations during inflation, which may capture the extra-dimensional structure of the Universe.[22]

Consider multi-field inflation with kinetic terms described by a metric  $G^{IJ}(\phi^K)$  in field space and  $n$  scalar fields. The action of the model can basically be described by

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} + P(X, \phi^I) \right], \quad (1.1)$$

where  $X$  is given by

$$X \equiv -\frac{1}{2} G_{IJ} \partial_\mu \phi^I \partial^\mu \phi^J. \quad (1.2)$$

For simplicity two-field inflation is considered in this paper, in which the adiabatic field  $\sigma$  and the entropy field  $s$  appear. Here the adiabatic and entropy fields are defined by  $\dot{\sigma}^2 \equiv \sum_I (\dot{\phi}^I)^2 = 2X$  and  $s$  ( $\dot{s} \equiv 0$ ). Using the kinetic part  $K$  and the potential  $V$ ,  $P(X, \phi^I)$  is expressed as  $P(X, \phi^I) = K(X, \phi^I) - V(\phi^I)$ .

Before discussing the details of the calculational method, which is based on the modulated inflaton velocity (“modulated inflation” in ref.[6]), it would be useful to show why in the  $\delta N$  formalism the modulated inflaton velocity can be used for the evolution equation of the curvature perturbation, instead of using the conventional non-adiabatic pressure perturbation or the time-derivative of the comoving curvature perturbation.

Following the traditional calculation [2], the spectrum of the curvature perturbation  $\mathcal{P}_{\mathcal{R}}(k)$  for the (adiabatic) inflaton field  $\sigma$  is given by

$$\mathcal{P}_{\mathcal{R}}(k) = \left( \frac{H}{\dot{\sigma}} \right)^2 \left( \frac{H}{2\pi} \right)^2, \quad (1.3)$$

where  $H$  is the Hubble parameter of the Universe and the right-hand side is evaluated at the epoch of the horizon exit  $k = aH$ . Here  $a$  is the cosmic-scale factor. In the above equation, the comoving curvature perturbation  $\mathcal{R}$  is considered, which can be identified with the curvature perturbation on uniform-density hypersurfaces  $\zeta$  ( $\mathcal{R} \simeq -\zeta$ ) by studying the evolution of  $\zeta$  at large scales. The gauge-invariant combinations for the curvature perturbation can be constructed as follows:

$$\begin{aligned} \zeta &= -\psi - H \frac{\delta \rho}{\dot{\rho}} \\ \mathcal{R} &= \psi - H \frac{\delta q}{\rho + p}, \end{aligned} \quad (1.4)$$

where  $\delta q = -\dot{\sigma} \delta \sigma$  is the momentum perturbation satisfying

$$\epsilon_m = \delta \rho - 3H \delta q, \quad (1.5)$$

where  $\epsilon_m$  is the perturbation of the comoving density. Linear scalar perturbations of a Friedman-Robertson-Walker(FRW) background were considered:

$$ds^2 = -(1 + 2A)dt^2 + 2a^2(t)\nabla_i B dx^i dt + a^2(t)[(1 - 2\psi)\gamma_{ij} + 2\nabla_i \nabla_j E]dx^i dx^j. \quad (1.6)$$

Here  $\rho$  and  $p$  denote the energy density and the pressure during inflation.

Note that in the traditional argument the evolution of the curvature perturbation at large scales is calculated to be given by the non-adiabatic pressure perturbation  $\delta p_{nad}$ ;

$$\dot{\zeta} \simeq -H \frac{\delta p_{nad}}{\rho + p}, \quad (1.7)$$

where  $\zeta$  and  $\mathcal{R}$  coincide ( $\zeta \simeq -\mathcal{R}$ ) at large scales and  $\delta p_{nad} \equiv \left[ \delta p - \frac{\dot{p}}{\dot{\rho}} \delta \rho \right]$  is a gauge-invariant perturbation.

In fact, from Eq.(1.4), it is found that

$$\dot{\zeta} = -\dot{\psi} - \frac{d}{dt} \left[ H \frac{\delta \rho}{\dot{\rho}} \right], \quad (1.8)$$

and the equations for the local conservation of the energy momentum lead to[23]

$$\dot{\delta \rho} = -3H(\delta \rho + \delta p) + (\rho + p) \left[ 3\dot{\psi} - \nabla^2(\sigma + v + B) \right], \quad (1.9)$$

where the scalar describing the shear is

$$\sigma = \dot{E} - B \quad (1.10)$$

and  $\nabla^i v$  is the perturbed 3-velocity of the fluid. Eq. (1.9) gives the equation for  $\dot{\psi}$ , which can be used to derive

$$\begin{aligned} \dot{\zeta} &\simeq -\frac{\dot{\delta \rho} + 3H(\delta \rho + \delta p)}{3(\rho + p)} + \frac{d}{dt} \left[ H \frac{\delta \rho}{3H(\rho + p)} \right] \\ &= -\frac{H}{\rho + p} \left[ \delta p - \frac{\dot{p}}{\dot{\rho}} \delta \rho \right] \\ &= -\frac{H}{\rho + p} \delta p_{nad} \end{aligned} \quad (1.11)$$

where  $\nabla^2(\sigma + v + B)$  is neglected.

Besides  $\dot{\zeta}$  defined above, it is useful to define the perturbed expansion rate with respect to the coordinate time<sup>2</sup>

$$\delta \tilde{\theta} \equiv -3\dot{\psi} + \nabla^2 \sigma, \quad (1.12)$$

which leads to<sup>3</sup>

$$\begin{aligned} \frac{d}{dt} \delta N &\equiv \frac{1}{3} \delta \tilde{\theta} \\ &\simeq -\dot{\psi} \\ &= \dot{\zeta} + \frac{d}{dt} \left( H \frac{\delta \rho}{\dot{\rho}} \right) \\ &= -\frac{H}{\rho + p} \delta p_{nad} + \frac{d}{dt} \left( H \frac{\delta \rho}{\dot{\rho}} \right). \end{aligned} \quad (1.13)$$

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<sup>2</sup>See Ref. [23, 24] for more details on the definitions.

<sup>3</sup> $\frac{d}{dt} \left[ H \frac{\delta \rho}{\dot{\rho}} \right]$  can be identified with the shear when the perturbation  $\delta \rho$  is defined on spatially flat slicing and it satisfies the adiabatic condition[23]. The shear at large scales is neglected in the above equation, but terms related to non-adiabatic perturbations are not disregarded.

Following the conventional definition of the  $\delta N$  formalism, **we choose the gauge whose slicing is flat at  $t_{ini}$  and uniform density at  $t$** . Using  $\zeta = -\psi$  on the specific choice of slice at  $t$ , the  $\delta N$  formula is given by

$$\zeta = \frac{1}{3} \int_{t_{ini}}^t \delta \tilde{\theta} dt = \delta N, \quad (1.14)$$

which shows that Eq. (1.11) and Eq.(1.13) are consistent, since the equation is for the curvature perturbation  $\zeta$  on uniform density slice at  $t$ . Here the equation for the perturbed expansion rate  $\delta \tilde{\theta}$  for the  $\delta N$  formula is practically valid in any gauge and slicing, but the relation between the curvature perturbation and  $\delta N$  is defined for the specific choice of slice at  $t$ .

We also define  $\dot{\zeta}_N$  in terms of the  $\delta N$  formalism defined for the uniform density hypersurfaces. From Eqs. (1.13) and (1.9)

$$\begin{aligned} \dot{\zeta}_N &\equiv \frac{d}{dt} \delta N = -\dot{\psi} \\ &\simeq -H \frac{\delta(\rho + p)}{(\rho + p)} - H \frac{\dot{\delta\rho}}{3(\rho + p)} \\ &\simeq -H \frac{\delta(\rho + p)}{(\rho + p)} \\ &= -H \frac{\delta(\dot{\sigma}^2)}{\dot{\sigma}^2}, \end{aligned} \quad (1.15)$$

where the adiabatic field  $\sigma$  is defined so that the action has the standard kinetic term.<sup>4</sup> Here  $\dot{\delta\rho}$  has been disregarded in the uniform density gauge. The basic formula of the evolution of  $\delta N$  is valid for any gauge and slicing, but the relation between  $\dot{\zeta}$  and  $\dot{\delta N}$  is defined in the specific slicing. As a result, in terms of the  $\delta N$  formalism, the evolution of the curvature perturbation can be explained using the perturbations related to the inflaton velocity ( $\delta(\dot{\sigma}^2)$ )[6]. In this paper, using the  $\delta N$  formalism defined for the uniform density hypersurfaces at  $t$ , a very simple method for calculating the evolution of the curvature perturbation is discussed.

## 2 Standard kinetic term

### The model

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<sup>4</sup>Due to the approximations that have been considered in deriving the equation, the result is not exact with regard to the shear perturbations that accompany  $k^2/a^2$  factor.

In order to explain the validity of the calculational method based on the modulated inflation scenario, and to explain how the method makes calculation very easy and clear, first the simplest model of multi-field inflation is considered, which is characterized by

$$P = X - V, \quad (2.1)$$

where the metric  $G^{IJ}$  in the definition of  $X$  is equivalent to the unit matrix (i.e. consider  $G_{IJ} = \delta_{IJ}$ ) and  $V(\phi^1, \dots, \phi^n)$  represents the scalar potential. The field equation derived from Eq. (1.1) with the condition (2.1) is

$$\ddot{\phi}^I + 3H\dot{\phi}^I + V_I = 0, \quad (2.2)$$

where  $V_I \equiv \frac{\partial V}{\partial \phi^I}$ , and the Hubble rate  $H$  is determined by the Friedman equation:

$$H^2 \equiv \frac{1}{3M_p^2} \left[ \frac{1}{2} \sum_I (\dot{\phi}^I)^2 + V \right], \quad (2.3)$$

where  $M_p$  is the reduced Planck mass. In a spatially flat FLRW space-time with metric given by

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2, \quad (2.4)$$

the scalar fields are homogeneous. In order to study cosmological fluctuations, the scalar perturbations of the metric are:

$$ds^2 = -(1 + 2A)dt^2 + 2aB_i dx^i dt + a^2[(1 - 2\psi)\delta_{ij} + 2E_{ij}]dx^i dx^j, \quad (2.5)$$

which leads to energy and momentum perturbations given in terms of the scalar field perturbations;

$$\delta\rho = \delta X + \delta V \quad (2.6)$$

$$\begin{aligned} 3H\delta q &= -3H \sum_I \dot{\phi}^I \delta\phi^I \\ &= \sum_I \left[ \ddot{\phi}^I + V_I \right] \delta\phi^I, \end{aligned} \quad (2.7)$$

where the last equation is obtained by using the field equation. Combining these equations, the comoving density perturbation is found to be given by

$$\begin{aligned} \epsilon_m &\equiv \delta\rho - 3H\delta q \\ &= \delta X + \left( \delta V - \sum_I V_I \delta\phi^I \right) - \sum_I \ddot{\phi}^I \delta\phi^I \\ &\equiv \delta X + \delta^{(2)}V - \sum_I \ddot{\phi}^I \delta\phi^I, \end{aligned} \quad (2.8)$$

which is a gauge-invariant quantity. Here  $\delta^{(2)}V$  denotes higher order corrections with respect to the field perturbations. The comoving density perturbation satisfies the evolution equation

$$\epsilon_m = -\frac{1}{4\pi G} \frac{k^2}{a^2} \Psi, \quad (2.9)$$

where  $\Psi$  is related to the shear perturbation.

### Calculation

The adiabatic and entropy fields are defined by  $\dot{\sigma}^2 \equiv \sum_I (\dot{\phi}^I)^2 = 2X$  and  $s$  ( $\dot{s} \equiv 0$ ). In terms of the adiabatic field, the momentum perturbation is given by

$$\delta q = -\dot{\sigma} \delta \sigma. \quad (2.10)$$

The comoving density perturbation in terms of the adiabatic and entropy fields is[25, 26]:

$$\begin{aligned} \epsilon_m &= \delta X - \ddot{\sigma} \delta \sigma + [\delta V - V_\sigma \delta \sigma] \\ &\simeq \delta X - \ddot{\sigma} \delta \sigma + V_s \delta s, \end{aligned} \quad (2.11)$$

where the term related to the change of the basis of the adiabatic field has been included in the definition of  $\delta X$ [27]. At large scales the equation leads to

$$\begin{aligned} \frac{\delta X}{X} &= \frac{\epsilon_m}{X} + \frac{\ddot{\sigma}}{X} \delta \sigma - \frac{V_s \delta s}{X} \\ &\simeq \frac{\ddot{\sigma}}{X} \delta \sigma - \frac{V_s \delta s}{X}, \end{aligned} \quad (2.12)$$

where the term proportional to  $\frac{k^2}{a^2} \Psi$  has been disregarded. Considering the perturbation of the inflaton velocity  $\frac{\delta(\dot{\sigma})^2}{\dot{\sigma}^2} = \frac{\delta X}{X}$  and the modulated inflation scenario,  $\dot{\zeta}_N$  is found to be

$$\begin{aligned} \dot{\zeta}_N &\simeq -H \frac{\delta X}{X} \\ &\simeq -2 \frac{V_s \delta s}{\dot{\sigma}^2}, \end{aligned} \quad (2.13)$$

where  $\ddot{\sigma} \delta \sigma / X$  has been neglected. For  $\mathcal{R} = -\delta N$ , it is found that

$$\dot{\mathcal{R}} \simeq -2 \frac{V_s \delta s}{\dot{\sigma}^2}. \quad (2.14)$$

Introducing a bend parameter  $\dot{\theta} \equiv -V_s / \dot{\sigma}$ , reveals[25]

$$\dot{\mathcal{R}} \simeq 2H \frac{\dot{\theta}}{\dot{\sigma}} \delta s. \quad (2.15)$$

In the above calculation the modulated inflaton velocity  $\delta(\dot{\sigma}) = 2\delta X$  has been obtained directly from the comoving energy density  $\epsilon_m$ . The calculation in terms of the modulated inflation scenario is thus very simple and straightforward compared with other calculations, which are based on the non-adiabatic pressure perturbation or the time-derivative of the comoving curvature perturbation on spatially flat slice.

The reason for  $\dot{\mathcal{R}} \neq 0$  is very clear in this scenario. The constancy of the curvature perturbation is violated due to the inhomogeneities of the inflaton velocity  $\delta(\dot{\sigma}^2) \neq 0$ , which is caused by the entropy field. Then the inhomogeneities of the inflaton velocity  $\delta(\dot{\sigma}^2) \neq 0$  causes inhomogeneities of the time spent during inflation, equivalently the inhomogeneities of the e-foldings  $\delta N_{mod} \neq 0$  before the end of inflation. The result is also useful for estimating the second order corrections from the potential. Introducing the quadratic potential  $\delta^{(2)}V = \frac{1}{2}m_s s^2$ , it is found that the second-order perturbation of the potential with respect to the field  $s$  leads to

$$\mathcal{R}^{(2)} \simeq -H \frac{m_s (\delta s)^2}{\dot{\sigma}^2}. \quad (2.16)$$

## 2.1 Non-equilibrium correction 1

The bend of the trajectory is important since the velocity perturbation  $\delta(\dot{\sigma})^2$  can become non-zero near the bend, even if it arises late after the horizon exit. At the bend of the trajectory, the velocity perturbation is given by

$$\lim_{k \rightarrow 0} \delta(\dot{\sigma}^2) \simeq \dot{\theta} \dot{\sigma} \delta s. \quad (2.17)$$

In addition to the inhomogeneities that may appear at large scales, there are small-scale (i.e., decaying) inhomogeneities of the inflaton velocity that can be related to the inhomogeneities in the slow-rolling velocity

$$\delta(\dot{\sigma}_{slow}) \equiv \delta \left( -\frac{V_{\sigma}}{3H} \right) \simeq -\frac{V_{\sigma,s}}{3H} \delta s, \quad (2.18)$$

which decays at large scales as  $\propto k^2/a^2$ [6]. The correction from such small-scale inhomogeneities can be dubbed non-equilibrium corrections, which may become additional source of the curvature perturbation. More specifically, Eq. (2.17) suggests that  $\delta(\dot{\sigma})^2$  soon decays to reach  $\delta(\dot{\sigma}^2) \simeq \dot{\theta} \dot{\sigma} \delta s$  after the horizon exit, and that at large scales the inflation



velocity perturbations do not lead to significant evolution  $\dot{\mathcal{R}} \neq 0$  except for the place where the inflation trajectory is curved. The velocity perturbation may have a decaying component accompanied by a significant decay factor  $k^2/a^2 \sim e^{-2Ht}$  [6]. In what follows we show why the corrections added by the decaying perturbations cannot be disregarded.

First, the small-scale inhomogeneities may affect the curvature perturbations at least just after horizon crossing. Then, we know that the perturbations added at small scales will be frozen at large scales. Therefore, if the small-scale corrections are significant, they can be observed in the present Universe. Note that the factor  $e^{-2Ht}$  in the integral with respect to the time coordinate does not always lead to strong suppression in the result. For the simplest example, the following integral is considered

$$\int_0^{t_e} \delta C H e^{-2Ht} dt \simeq \frac{1}{2} \delta C, \quad (2.19)$$

which is such that no exponential suppression remains after integration.

To show explicitly the significant effect from the decaying component in the modulated inflation velocity, here a very simple mechanism is considered to add a large non-Gaussian effect to the conventional inflationary perturbation. Consider the standard hybrid-type inflation with a non-standard interaction with addition scalar fields  $\chi_i$ ;

$$V_{int} \sim g^2 v_0 |\phi - \phi_{ESP}| \chi_i^2, \quad (2.20)$$

where  $v_0$  denotes an intermediate mass scale. The inflaton potential during inflation is given by

$$V(\phi) = m_\phi^2 \phi^2 + V_0. \quad (2.21)$$

In this model the adiabatic inflaton field is  $\phi$ . Enhanced symmetric point (ESP) at  $\phi = \phi_{ESP}$  is defined as the point where the scalar fields  $\chi_i$  become (temporarily) massless during inflation. The inhomogeneities in  $V_{int}$  caused by the light fields  $\chi_i$  lead to the small-scale velocity perturbation

$$\delta \dot{\phi} \simeq \frac{n g^2 v_0 (\delta \chi)^2}{3H}, \quad (2.22)$$

where  $n$  is the number of massless fields at the ESP. Note that there is no bend in the classical (unperturbed) trajectory in this model. The small-scale inhomogeneities of the inflaton velocity thus lead to the correction given by

$$\Delta R \equiv \int \dot{\mathcal{R}} dt \simeq 2 \int H \frac{\delta \dot{\phi}}{\dot{\phi}} e^{-2H(t-t_{ESP})} dt, \quad (2.23)$$

which can be very large when the inflaton passes through the ESP at  $t = t_{ESP}$ . Considering the standard curvature perturbations  $\mathcal{R}_0 \sim H\delta\phi/\dot{\phi}$ , we find

$$\begin{aligned}\Delta R &\sim \frac{\delta\dot{\phi}}{\dot{\phi}} \\ &\sim \frac{ng^2v_0(\delta\chi)^2}{3H^2\delta\phi}\mathcal{R}_0.\end{aligned}\tag{2.24}$$

It is useful to specify the level of non-Gaussianity by the non-linear parameter  $f_{NL}$ ,<sup>5</sup> which is usually defined by the Bardeen potential  $\Phi$ ,

$$\Phi = \Phi_{Gaussian} + f_{NL}\Phi_{Gaussian}^2.\tag{2.25}$$

Using the Bardeen potential, the curvature perturbation  $\zeta$  is given by

$$\Phi = \frac{3}{5}\zeta.\tag{2.26}$$

When we consider “additional” non-Gaussianity created at the ESP, the first-order perturbation is generated dominantly by the usual inflaton perturbation  $\delta\phi$ . Therefore, the second-order perturbation is not correlated to the first-order perturbation. In this case,  $\zeta$  can be expanded by the  $\delta N$  formalism as

$$\zeta \simeq N_\phi\delta\phi + N_\chi\delta\chi + \frac{1}{2}N_{\phi\phi}\delta\phi^2 + \frac{1}{2}N_{\chi\chi}\delta\chi^2 + \dots,\tag{2.27}$$

and we assume that the perturbation can be separated as

$$\zeta = \zeta^{(\phi)} + \zeta^{(\sigma)}.\tag{2.28}$$

The calculation of the non-linear parameter  $f_{NL}$  for the uncorrelated perturbations  $\delta\chi$  and  $\delta\phi$  is discussed in ref.[28], where a useful simplification for is

$$f_{NL} \simeq \left( \frac{1}{1300} \frac{N_{\sigma\sigma}}{N_\phi^2} \right)^3,\tag{2.29}$$

where  $\delta\chi \sim \delta\phi$  is assumed for simplicity. Therefore, the non-linear parameter for the present model is estimated as

$$f_{NL} \sim \left( \frac{\Delta R}{1300\mathcal{R}_0^2} \right)^3 \sim \left( \frac{ng^2v_0}{10^3\mathcal{R}_0H} \right)^3.\tag{2.30}$$

The important suggestion from the model is that **a significant scale-dependence may arise for the non-linear parameter  $f_{NL}(k)$**  at a scale corresponding to  $\phi = \phi_{ESP}$  [29], where the  $\chi_i$  fields become massless at the horizon exit. We show a typical situation in Fig. 1. Since ESPs may typically appear in a brane Universe, there is a hope that we may

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<sup>5</sup>See ref.[1] for more details.

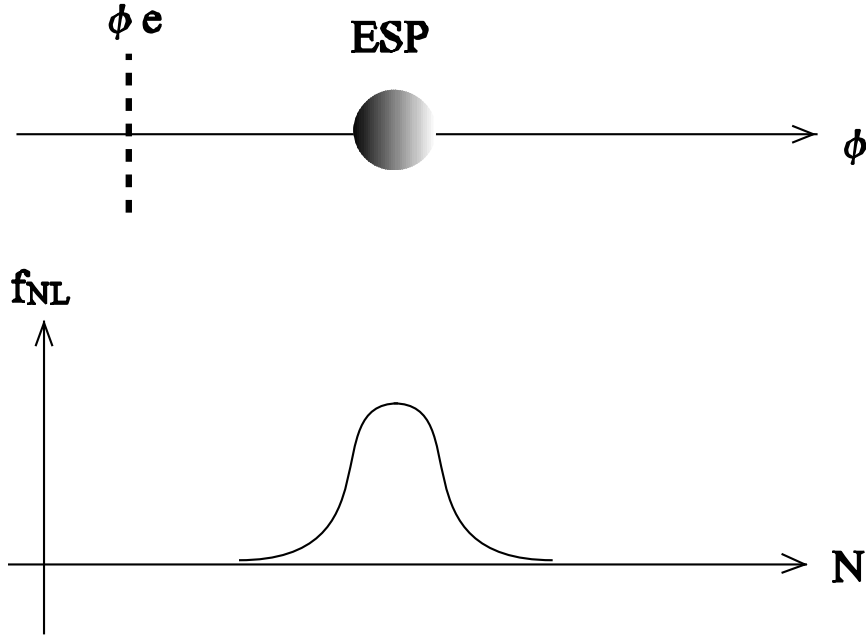


Figure 1: In a brane inflation model one may find an enhanced symmetric point on the inflation trajectory, where massless excitations (open string modes) may arise[29]. If the massless excitations are coupled to the inflaton, the fluctuations of the massless mode may cause fluctuations of the inflaton velocity, which adds a significant non-Gaussian character to the spectrum.

scan the moduli or even find the signature of extra dimensions from the consideration of the scale-dependence in the cosmological perturbations [22, 30, 29].

Again, what is important in this argument is that decaying-components of the velocity perturbation may cause significant anomalies in the spectrum, which may be seen in some non-linear parameter or in some other cosmological parameters as a signature of massless excitations during inflation.

Before closing this section, it would be useful to compare our model with a model with a step (or a spike) in the potential[31]. A step in the potential leads to a time-dependent slow-roll parameters, which causes  $f_{NL} \neq 0$  for a single-field inflation model. In this case the origin of the higher order perturbations are terms proportional to  $\epsilon$  or  $\eta$ , which are not important in the smooth potential but can cause significant non-gaussianity at the step. The obvious discrepancy between the model with step and our model with ESP is that in the former model the origin of the non-gaussianity is  $\epsilon \neq 0$  or  $\eta \neq 0$ , while in the latter

model the origin is the second-order perturbation in the velocity. Therefore, these two effects appear separately in the calculation and give independent results for non-linear parameters. In fact, in the present model  $\dot{\epsilon}$  and  $\dot{\eta}$  at the 0-th order are very small at the ESP.

## 2.2 Non-equilibrium correction 2

In addition to the non-equilibrium correction from the modulated inflaton velocity, there can be significant correction from  $\ddot{\sigma}$  when deviation from slow-roll is significant[32, 33].  $\ddot{\sigma}$  in the equation of  $\dot{\mathcal{R}}$  is essential in explaining the evolution of  $\mathcal{R}$  when the deviation is significant. Allowing a short period of deviation from the slow-roll, the most significant effect may occur when the inflaton temporarily stops. Expressing the conventional curvature perturbation for slow-roll inflation as  $\mathcal{R} \simeq -\frac{H}{\dot{\sigma}}\delta\sigma$ , the divergence is brought about by  $\dot{\sigma} \simeq 0$  in the denominator. To understand the correction from  $\ddot{\sigma}$  when the inflaton stops, we split the inflaton velocity as

$$\dot{\sigma} = \dot{\sigma}_s + \dot{\sigma}_d, \quad (2.31)$$

where  $\dot{\sigma}_s$  satisfies the slow-roll condition  $\dot{\sigma}_s = -V_\sigma/3H$ . Considering the equation

$$\ddot{\sigma} + 3H[\dot{\sigma}_s + \dot{\sigma}_d] + V_\sigma = 0, \quad (2.32)$$

the decaying velocity  $\dot{\sigma}_d$  follows  $\ddot{\sigma} = -3H\dot{\sigma}_d$ . Replacing  $\ddot{\sigma}$  in Eq.(2.11) by  $-3H\dot{\sigma}_d$ , it is found for  $\dot{\theta} = 0$ ;

$$\begin{aligned} \dot{\mathcal{R}} &\simeq -6H \left( \frac{\dot{\sigma}_d}{\dot{\sigma}} \right) \left( H \frac{\delta\sigma}{\dot{\sigma}} \right) \\ &= -6H\epsilon_d^{-1}\mathcal{R}. \end{aligned} \quad (2.33)$$

Here a parameter  $\epsilon_d \equiv \dot{\sigma}/\dot{\sigma}_d$  is introduced, which leads to  $\epsilon_d^{-1} \rightarrow \infty$  when the inflaton stops temporarily during inflation. It is found from the equation that the curvature perturbation, which may become anomalously large when the inflaton stops, will decay rapidly as  $\propto \exp[-3H\epsilon_d^{-1}t]$ . See also Ref.[32] for discussions in terms of the usual perturbation theory, and Ref.[33] for another discussion in terms of the  $\delta N$  formalism.

### 3 Simple extension

#### The model

Our second example is characterized by an extension of the inflation kinetic term with a metric for the field space:

$$G_{\phi\phi} = \omega^A(\phi, \chi), \quad G_{\chi\chi} = \omega^B(\phi, \chi), \quad (3.1)$$

where  $\omega^A$  and  $\omega^B$  are functions determined by the scalar fields. For simplicity, the case with the diagonal metric is considered. The explicit form of the action is given by

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} M_p^2 \mathcal{R} - \frac{\omega^A}{2} (\partial_\mu \phi) (\partial^\mu \phi) - \frac{\omega^B}{2} (\partial_\mu \chi) (\partial^\mu \chi) - V(\phi, \chi) \right]. \quad (3.2)$$

The field equations are

$$\omega^A \ddot{\phi} + \dot{\omega}^A \dot{\phi} + 3\omega^A H \dot{\phi} + V_\phi - \frac{1}{2} \omega_\phi^B \dot{\chi}^2 = 0 \quad (3.3)$$

$$\omega^B \ddot{\chi} + \dot{\omega}^B \dot{\chi} + 3\omega^B H \dot{\chi} + V_\chi - \frac{1}{2} \omega_\chi^A (\dot{\phi})^2 = 0. \quad (3.4)$$

The comoving density perturbation is found to be

$$\begin{aligned} \epsilon_m &\equiv \delta\rho + 3HG_{IJ}\dot{\phi}^J\delta\phi^I \\ &= \delta X - (\omega^A \ddot{\phi} \delta\phi + \omega^B \ddot{\chi} \delta\chi) + [\delta V - V_\chi \delta\chi - V_\phi \delta\phi] \\ &\quad + \left( \frac{\omega_\phi^B}{2} \delta\phi (\dot{\chi})^2 + \frac{\omega_\chi^A}{2} \delta\chi (\dot{\phi})^2 \right) - \dot{\omega}^A \dot{\phi} \delta\phi - \dot{\omega}^B \dot{\chi} \delta\chi \\ &\simeq \delta X + \left( \frac{\omega_\phi^B}{2} \delta\phi (\dot{\chi})^2 + \frac{\omega_\chi^A}{2} \delta\chi (\dot{\phi})^2 \right) - \dot{\omega}^A \dot{\phi} \delta\phi - \dot{\omega}^B \dot{\chi} \delta\chi. \end{aligned} \quad (3.5)$$

The adiabatic field in this model is defined by  $\dot{\sigma}^2/2 \equiv X$ .

#### Calculation

The velocity perturbation for the adiabatic field  $\dot{\sigma}^2 = \omega^A \dot{\phi}^2 + \omega^B \dot{\chi}^2$  is given by

$$\frac{\delta X}{X} \simeq \frac{\epsilon_m - \left( \frac{\omega_\phi^B}{2} \delta\phi (\dot{\chi})^2 + \frac{\omega_\chi^A}{2} \delta\chi (\dot{\phi})^2 \right) + \dot{\omega}^A \dot{\phi} \delta\phi + \dot{\omega}^B \dot{\chi} \delta\chi}{X}. \quad (3.6)$$

Using the modulated inflation scenario, the form of  $\dot{\mathcal{R}}$  in terms of the original fields  $\phi$  and  $\chi$  is found to be

$$\dot{\mathcal{R}} \simeq H \left( \frac{\omega_\phi^B}{2X} \delta\phi (\dot{\chi})^2 + \frac{\omega_\chi^A}{2X} \delta\chi (\dot{\phi})^2 \right) - H \frac{\dot{\omega}^A \dot{\phi} \delta\phi + \dot{\omega}^B \dot{\chi} \delta\chi}{X}, \quad (3.7)$$

which gives the evolution of the curvature perturbation in the slice defined for  $\dot{\zeta}_N$ .

In terms of the adiabatic field  $\sigma$  and the entropy field  $s$ , the action is precisely the same as the model discussed in Sec.2. Therefore, the evolution equation for the curvature perturbation is given by

$$\dot{\mathcal{R}} \simeq -2H \frac{V_s}{\dot{\sigma}^2} \delta s, \quad (3.8)$$

which is precisely the same as the two-field inflation model with the standard kinetic term.

## 4 Modulated inflation for a generalized multi-field inflation

### The model

We consider multi-field inflation with kinetic terms with a metric  $G^{IJ}(\phi^L)$  in field space. The original action described by  $\phi^I$  is;

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} + P(X, \phi^I) \right], \quad (4.1)$$

where  $X$  is given by

$$X \equiv -\frac{1}{2} G_{IJ} \partial_\mu \phi^I \partial^\mu \phi^J. \quad (4.2)$$

In this section, a separation is considered

$$P(X, \phi^I) = K(X, \phi_I) - V(\phi^I) \quad (4.3)$$

and set  $8\pi G = 1$  for simplicity. In what follows we use the basic equations given in Ref.[34]. The field equation is given by

$$\ddot{\phi}^J + \left( 3H + \frac{\dot{G}_{IJ}}{G_{IJ}} + \frac{\dot{K}_X}{K_X} \right) \dot{\phi}^J - \frac{K_{[I} - V_I}{K_X G_{IJ}} = 0. \quad (4.4)$$

The definition of the derivative  $K_{[I}$  may be somewhat confusing. We introduce two different definitions for the derivatives

$$\begin{aligned} K(X, \phi_I)_{[I]} &\equiv K_X X_I - K_I \\ K(X, \phi_I)_I &\equiv \frac{\partial K}{\partial \phi^I}. \end{aligned} \quad (4.5)$$

The energy density for this action is given by

$$\rho = 2XK_X - K + V, \quad (4.6)$$

which leads to the Friedman equation

$$H^2 = \frac{1}{3}(2XK_X - K + V) \quad (4.7)$$

and the time-derivative of the Hubble parameter

$$\dot{H} = -XK_X. \quad (4.8)$$

Combining the above equation with the continuity equation  $\dot{\rho} = -3H(\rho + p)$ , it is found that

$$\rho + p = -2\dot{H}. \quad (4.9)$$

### Calculation

For generalized multi-field inflation, the natural definition of the adiabatic field  $\tilde{\sigma}$  is

$$\dot{\tilde{\sigma}}^2 \equiv \rho + p = 2XK_X, \quad (4.10)$$

which recovers the basic equation (1.15) in terms of  $\tilde{\sigma}$ . The definition is very natural and consistent with the intrinsic property of the adiabatic field. However, in past studies a more simple definition  $\dot{\sigma} \equiv \sqrt{2X}$  has been considered. The discrepancy caused by the definition of the adiabatic field may lead to an error in the result.

In the followings, in addition to the natural definition of the adiabatic field  $\tilde{\sigma}$ , the familiar definition  $\dot{\sigma} \equiv \sqrt{2X}$  is also considered so that these results from different definitions of the adiabatic field can be compared.

The velocity perturbation for the adiabatic field  $\dot{\tilde{\sigma}}^2 \equiv 2XK_X$  is given by

$$\frac{\delta(\dot{\tilde{\sigma}}^2)}{\dot{\tilde{\sigma}}^2} = \frac{\delta X}{X} + \frac{\delta K_X}{K_X}, \quad (4.11)$$

whereas for the usual definition  $\dot{\sigma}^2 \equiv 2X$ , it is found that

$$\frac{\delta(\dot{\sigma}^2)}{\dot{\sigma}^2} = \frac{\delta X}{X}. \quad (4.12)$$

The definition of the adiabatic field is important. Using the adiabatic field and the  $\delta N$  formalism,  $\dot{\mathcal{R}}$  is calculated from Eq.(1.15) for the adiabatic field defined by  $\tilde{\sigma}$ , not by  $\sigma$ . Obviously, the discrepancy between Eqs.(4.11) and (4.12) is crucial for the calculation. In order to calculate  $\dot{\mathcal{R}}$  in terms of the  $\delta N$  formalism, the explicit form of  $\delta X$  at large scales needs to be found. As in the former arguments for simpler models in Sect.2 and 3, the explicit form of the velocity perturbation is obtained from the perturbation of the comoving energy density. The perturbation of the comoving energy density for the generalized multi-field inflation model is

$$\epsilon_m \equiv \delta\rho + 3HK_X G_{IJ} \dot{\phi}^J \delta\phi^I, \quad (4.13)$$

where the energy density perturbation is given by

$$\begin{aligned} \delta\rho &= \delta X (K_X + 2XK_{XX}) + \delta\phi^I (2XK_{XI} - K_I + V_I) + 2X\delta^{(2)}K_X - \delta^{(2)}P \\ &\simeq \frac{K_X}{c_s^2} \delta X + \delta\phi^I (2XK_{XI} - K_I + V_I), \end{aligned} \quad (4.14)$$

where the higher order perturbations  $\delta^{(2)}K_X$  and  $\delta^{(2)}P$  has been disregarded in the last equation. Here the effective sound speed defined by  $c_s^2 \equiv K_X/(K_X + 2XK_{XX})$  is introduced. Considering the expansion of the time-derivative, we find

$$\begin{aligned} \dot{K}_X &= K_{XX}\dot{X} + K_{XL}\dot{\phi}^L \\ &= K_{XX} \left( \frac{\dot{G}_{IJ}}{G_{IJ}} X + \frac{1}{2} G_{IJ} \ddot{\phi}^I \dot{\phi}^J + \frac{1}{2} G_{JI} \ddot{\phi}^J \dot{\phi}^I \right) + K_{XL} \dot{\phi}^L, \end{aligned} \quad (4.15)$$

which is used to rewrite the field equation as

$$\frac{1}{c_s^2} \ddot{\phi}^I + \left[ 3H + \left( 1 + \frac{XK_{XX}}{K_X} \right) \frac{\dot{G}_{IJ}}{G_{IJ}} + \frac{K_{XL}\dot{\phi}^L}{K_X} \right] \dot{\phi}^I - \frac{G^{IJ}}{K_X} (K_{[J} - V_J) = 0. \quad (4.16)$$

Using the field equation, the perturbation of the comoving energy density is found to be given by

$$\begin{aligned} \epsilon_m &\simeq \frac{K_X}{c_s^2} \delta X + \delta\phi^I (2XK_{XI} - K_I + V_I) + 3HK_X G_{IJ} \dot{\phi}^J \delta\phi^I \\ &\simeq \frac{K_X}{c_s^2} \delta X + \delta\phi^I \left( -K_I + K_{[I} - \dot{G}_{IJ} \dot{\phi}^J [K_X + XK_{XX}] \right). \end{aligned} \quad (4.17)$$

We basically followed the calculation in Sec.2 and Sec.3. In order to calculate the time-



derivative of the metric for the field space, the following expansion is considered<sup>6</sup>

$$\begin{aligned}\dot{G}_{IJ}\dot{\phi}^J &= \left[\partial_L G_{IJ}\dot{\phi}^L\right]\dot{\phi}^J \\ &= \frac{\partial_L G_{IJ}}{G_{JL}}2X,\end{aligned}\tag{4.18}$$

which leads to  $\epsilon_m$  given by

$$\epsilon_m \simeq \frac{K_X}{c_s^2}\delta X + \delta\phi^I \left(-K_I + K_{[I]} - (K_X + XK_{XX})2X\frac{\partial_L G_{IJ}}{G_{JL}}\right).\tag{4.19}$$

Here the difference between  $K_{[I]}$  and  $K_I$  is crucial for the model discussed in Sec.3.

It would be useful to calculate  $\dot{\mathcal{R}}$  from the usual definition of the adiabatic field  $\sigma$  using the modulated inflaton velocity. Expressing the action in terms of the adiabatic and entropy fields,  $G_{\sigma s}$  and  $G_{ss}$  in the above equation must vanish. The comoving density perturbation is thus given by[34]

$$\begin{aligned}\epsilon_m &\simeq \frac{K_X}{c_s^2}\delta X + \delta s(2XK_{Xs} - P_s) + \delta\sigma(2XK_{X\sigma} - P_\sigma) + 3HK_X G_{\sigma\sigma}\dot{\sigma}\delta\sigma \\ &\simeq \frac{K_X}{c_s^2}\delta X + \delta s(2XK_{Xs} - P_s) \\ &\quad + \delta\sigma \left[-P_\sigma + P_{[\sigma]} - (K_X + XK_{XX})2X\frac{\partial_\sigma G_{\sigma\sigma}}{G_{\sigma\sigma}}\right] \\ &\quad - (K_X + 2XK_{XX})G_{\sigma\sigma}\ddot{\sigma}\delta\sigma \\ &\simeq \frac{K_X}{c_s^2}\delta X + \delta s(2XK_{Xs} - P_s),\end{aligned}\tag{4.20}$$

where terms proportional to  $\delta\sigma$  disappeared using  $G_{\sigma\sigma} = 1$  and  $\ddot{\sigma} \simeq 0$ . To find the velocity perturbation, we need to find  $\delta X$  from the comoving energy density. The perturbation  $\delta X$  caused by the entropy perturbation  $\delta s$  is given by

$$\frac{\delta X}{X} \simeq -\frac{c_s^2}{XK_X}[(2XK_{Xs} - P_s)\delta s].\tag{4.21}$$

where the adiabatic field is defined by  $\dot{\sigma} \equiv \sqrt{2X}$ .<sup>7</sup>

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<sup>6</sup>The simplification is not valid when  $G_{JL} = 0$ . The original equation must be used for  $G_{JL} = 0$ .

<sup>7</sup>With regard to our definition of  $\delta X$ , the change of the basis of the adiabatic field is already included in the definition. However, if one redefines the evolution of the curvature perturbation by  $\hat{\zeta}_N \equiv -H\frac{\delta\dot{\sigma}}{\dot{\sigma}} \equiv -H\left[\frac{\delta X}{2X} + \frac{\dot{e}_\sigma^I \delta\phi^I}{\dot{\sigma}}\right]$ , where  $\dot{e}_\sigma^I = P_s e_s^I / P_X \dot{\sigma}$  is the rate of change of the adiabatic basis vector  $e_\sigma^I$  in terms of the entropy basis vector  $e_s^I$ , it leads to  $\dot{\mathcal{R}} \simeq \frac{H}{2XP_X} [(1 + c_s^2)P_s \delta s - 2Xc_s^2 K_{Xs} \delta s]$ . Compared with the result and the calculation presented in the previous study,  $\dot{\mathcal{R}}$  in Ref.[34] is reproduced by  $\hat{\mathcal{R}}$  redefined above.

Here the useful expression for  $\dot{\mathcal{R}}$  is written in terms of the original fields  $\phi^I$  or the adiabatic field defined by  $\sigma$ . The calculation based on  $\sigma$  is useful when  $X$  has important meaning in the string theory. Our result is

$$\begin{aligned}
\dot{\mathcal{R}} &\simeq H \frac{\delta(\dot{\tilde{\sigma}}^2)}{\dot{\tilde{\sigma}}^2} \\
&= \left[ \frac{\delta X}{X} + \frac{\delta K_X}{K_X} \right] \\
&= H \left[ \frac{\delta X}{X} + \frac{\delta X K_{XX} + \delta \phi^I K_{XI}}{K_X} \right] \\
&= H \left[ \frac{\delta X}{X} \left( \frac{K_X + X K_{XX}}{K_X} \right) + \frac{\delta \phi^I K_{XI}}{K_X} \right] \\
&\simeq -H \frac{\tilde{c}_s^2 (2X K_{Xs} - P_s) \delta s}{X K_X} + H \frac{K_{Xs}}{K_X} \delta s,
\end{aligned} \tag{4.22}$$

where  $\tilde{c}_s^2$  is defined by

$$\tilde{c}_s^2 \equiv c_s^2 \frac{K_X + X K_{XX}}{K_X} = \frac{K_X + X K_{XX}}{K_X + 2X K_{XX}}. \tag{4.23}$$

This result is useful in practical situations where  $\dot{\mathcal{R}}$  would be calculated in terms of  $\sigma$ . For the original fields  $\phi^I$ , it can be found that

$$\begin{aligned}
\dot{\mathcal{R}}_{\tilde{\sigma}} &\equiv H \frac{1}{X K_X} [(K_X + X K_{XX}) \delta X + X K_{XI} \delta \phi^I] \\
&\simeq \frac{\tilde{c}_s^2 H}{X K_X} \left[ (K_I - K_{[I])} \delta \phi^I + (K_X + X K_{XX}) 2X \frac{\partial_L G_{IJ}}{G_{JL}} \delta \phi^I \right] \\
&\quad + H \frac{K_{XI}}{K_X} \delta \phi^I.
\end{aligned} \tag{4.24}$$

## 5 Conclusions and discussions

In this paper the time-dependence of the curvature perturbation is considered in terms of the  $\delta N$  formalism. What is new in this study is (1) the  $\delta N$  calculation in terms of the modulated inflation velocity (2) the explicit form of  $\dot{\mathcal{R}}$  calculated with respect to the adiabatic field  $\tilde{\sigma}$ . Our method is new and very simple, which can be used to understand more exciting topics including the evolution during warm inflation[35] and the evolution for the generalized gravity theory[36].

Our last comment is for the importance of the non-equilibrium corrections. As we have discussed in this paper, there are many kinds of decaying inhomogeneities that may

not be disregarded. For example, the inhomogeneities caused by the inflaton velocity  $\delta(\dot{\sigma}_{slow}) \equiv \delta\left(-\frac{V_{\sigma}}{3H}\right)$  is not significant at large scales, however after time integration the small-scale inhomogeneities may leave significant correction to the curvature perturbation, and the correction will be fixed at large scales due to the constancy of the curvature perturbation.

In addition to the correction induced by the decaying inhomogeneities,  $\ddot{\sigma}\delta\sigma$  may lead to significant variation of  $\mathcal{R}$ . Namely, one can find a significant result when the inflaton stops temporarily during inflation, where the curvature perturbation can be separated into (initially very large) decaying component and (standard) non-decaying components[32]. It is possible to extract the non-decaying component of the curvature perturbation by using the  $\delta N$  formalism [33]; however the evolution with deviation is not clearly understood in more general situations.

The above two corrections may be dubbed non-equilibrium corrections, and may be important in string cosmological models. It is important to understand the evolution of the curvature perturbation in terms of the non-equilibrium corrections.

Our hope is to understand the effect of massless excitations and non-equilibrium dynamics during inflation, so that we can find signatures of inflaton potential and extra-dimensional structure in the sky.

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